NEW CONCEPTS FROM THE EVANS UNIFIED FIELD THEORY, PART TWO: DERIVATION OF THE HEISENBERG EQUATION AND REPLACEMENT OF THE HEISENBERG UNCERTAINTY PRINCIPLE.

by

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ABSTRACT

The Heisenberg equation of motion is derived from the Evans wave equation of motion of generally covariant unified field / matter theory as the non-relativistic quantum limit of general relativity. The method used is first to derive the free particle Klein Gordon wave equation in the special relativistic limit of the Evans wave equation. The free particle Einstein equation for special relativistic motion (the equation of special relativistic momentum) is found in the classical limit of the Klein Gordon equation, together with the fundamental operator equivalence of quantum mechanics. The free particle Newton equation is found as the non-relativistic limit of the Einstein equation of special relativity (the relativistic momentum equation) by using the definition of relativistic kinetic energy. The time-independent free particle Schrödinger equation is found from the already derived operator equivalence applied to Newton's equation. Finally the Heisenberg equation is found as a form of the Schrödinger equation. The self consistency of the method used is checked with the rotational form of the Evans field equation. The well known commutator relation which is the basis of the acausal and therefore subjective "uncertainty" principle is derived from causal and therefore objective general relativity using the geometrical concepts of tetrad and torsion. The Planck constant is defined as the minimum quantity of action, or angular

momentum, allowed by the Evans Principle of Least Curvature, which together with the Evans Lemma and wave equation, give all the known principles of quantum mechanics from causal general relativity, and also suggest several new laws of nature from which new engineering can evolve and in some sectors, has been implemented already.

Keywords: Evans unified field theory, generally covariant unified field theory, Evans wave equation, Evans field equation, Evans Lemma, Heisenberg equation, the Evans principle of least curvature.

1.INTRODUCTION.

The Heisenberg equation {1} is the basis of much of quantum mechanics and matrix mechanics in the non-relativistic limit. In this paper, the second of a series {2} on new concepts from Evans' generally covariant unified field / matter theory {3-12}, the Heisenberg equation is derived self consistently both from the Evans wave equation and the Evans field equation in rotational form. The acausal and subjective uncertainty principle of Heisenberg is replaced by a causal and objective law from general relativity. This new law has the same mathematical structure as the uncertainty principle but is interpreted using the well known tetrad and torsion forms of differential geometry {2-13}. The complicated twentieth century dialogue between general relativity and quantum mechanics is therefore resolved straightforwardly by realizing that the wave-function originates in non-Euclidean spacetime which is in general can produce both curvature and torsion. The key to this philosophical resolution is the Evans Lemma {2-12}, which is the subsidiary proposition of the Evans wave equation, and which asserts that:

$$\Box q^a_\mu = R q^a_\mu \qquad -(1)$$

where q^a_μ is the tetrad, and R a scalar curvature. Scalar curvature R is therefore the eigenvalue of the d'Alembertian, $\Box$, operating on the tetrad, q^a_μ, as eigenfunction. This quantization of scalar curvature leads directly to the Evans wave equation:

$$(\Box + kT) q^a_\mu = 0 \qquad -(2)$$

using the fundamental Einsteinian relation:

$$R = -kT \qquad -(3)$$

where k is the Einstein constant of general relativity and where T is the index contracted canonical energy-momentum tensor {14}.

Quantum mechanics is therefore obtained straightforwardly from general relativity by recognising that the wavefunction is the tetrad. The latter is defined for any vector V^μ by:

$$V^a = q^a_\mu V^\mu \qquad -(4)$$

where a is the index of the orthonormal spacetime {13}of the base manifold indexed μ . The Lemma (1) and wave equation (2) are then direct mathematical consequences {2-12} of the well known and fundamental {13} tetrad postulate of differential geometry:

$$D_\mu q^a_\mu = 0 \qquad -(5)$$

where D_μ is the covariant derivative. The tetrad is essentially the matrix linking two frames of reference, one of which {13} is orthogonal and normalized, i.e. is a Euclidean or flat spacetime, and the other is the non-Euclidean base manifold. The flat spacetime is labelled a and the base manifold is labelled μ. The vector V^a can be defined using a representation space of any dimension, and the basis set a can be defined in any appropriate way {13}. The tetrad postulate, Lemma and wave equation are true for any dimension of representation space and any basis representation {2-12} (e.g. unit vectors or Pauli matrices). Therefore the Dirac equation and chromodynamics can be derived from the Evans equation by appropriate definition of the tetrad. There are only four physically significant dimensions however, time and three space dimensions. This means that the Evans unified field theory has a great advantage of simplicity over string theory (with its many spurious, unphysical "dimensions"). The Evans theory has been tested extensively against data {2-12} and has produced new engineering in several sectors {14}. String theory has produced nothing because it is an unphysical theory. Similarly the Heisenberg uncertainty principle can be of no interest to engineers, or in fact, to physicists. There are many other examples of spurious concepts in the history of science, for example the Aristotelian epicycles which preceded the Kepler laws and the great Newtonian synthesis of 1665.

In Section 2 the Heisenberg equation is derived as a quantum non-relativistic limit of the Evans wave equation (2). The derivation is checked using the rotational form of the Evans field equation. In Section 3 the Heisenberg uncertainty principle is discarded in favor of a new commutator law built up straightforwardly from differential geometry and the Evans principle of least curvature {2-12}. The latter identifies the Planck constant as the least unit of action or angular momentum in general relativity. The new law has the same mathematical structure as the older uncertainty principle, but is causal and is derived from general relativity. It is therefore objective in the sense that all physical laws must be objective, must predict events using mathematics, and must be objectively measurable against data. Heisenberg and the Copenhagen School introduced spurious concepts which are not useful in natural philosophy. The most obscure of these is the assertion that certain events are "unknowable" {1}. As soon as this assertion is made, however, the boundaries of natural philosophy are breached, because any assertion on the unknowable is subjective or theistic. Natural philosophy measures data objectively. The Copenhagen School asserts that the measuring process affects data in an acausal or unknowable way. General relativity is diametrically at odds with this assertion, and within general relativity measurement is objective: any event has a measurable cause. Recently {15} copious experimental evidence has been published which shows that the uncertainty principle is, unsurprisingly, untenable.

The Heisenberg EQUATION on the other hand is simply a restatement of the slightly earlier Schrödinger equation, and hardly deserves an appellation. It is given one because of the spurious uncertainty principle whose structure is derived from the form of the Heisenberg equation, or more properly, from the Schrödinger equation and de Broglie wave particle dualism. So in this paper we are at pains to separate the notion of uncertainty principle from our unified field theory. We CAN derive the mathematical form of the uncertainty principle, but do not interpret it in the manner of the Copenhagen School.

2. DERIVATION OF THE HEISENBERG EQUATION.

In its simplest form {1} the Heisenberg equation is:

$$(xp - px)\psi = i\hbar\psi \qquad -(6)$$

where:

$$p = -i\hbar\frac{\partial}{\partial x} \qquad -(7)$$

is a differential operator representing momentum, and where x represents position. Here ψ is the wavefunction or eigenfunction and $\hbar$ is the reduced Planck constant ($h/2\pi$). Eq. (6) is often stated as:

$$xp - px = i\hbar \quad -(8)$$

and was spuriously elevated by Heisenberg into a principle {1} which has come to be known as the Heisenberg uncertainty principle, rejected by Einstein and the deterministic School. The operators in Eq. (7) are defined as follows:

$$xp\psi = -ix\hbar\frac{\partial\psi}{\partial x} \quad -(9)$$

$$px\psi = -i\hbar\frac{\partial}{\partial x}(x\psi) = -i\hbar\left(x\frac{\partial\psi}{\partial x} + \psi\right). \quad -(10)$$

Note that p operates on the product $x\psi$ using the Leibnitz Theorem. From Eqs. (9) and (10):

$$(xp - px)\psi = -i\hbar x\frac{\partial\psi}{\partial x} + i\hbar x\frac{\partial\psi}{\partial x} + i\hbar\psi = i\hbar\psi. \quad -(11)$$

The method used to derive Eq. (7) from Eq. (2) is to first derive the time-independent Schrödinger equation {1} in the appropriate limit, then recognise that the Heisenberg equation is a restatement of the Schrödinger equation using the operator equivalence of quantum mechanics, an example of which is Eq. (7). The relativistic form of the operator equivalence is DERIVED from the Evans wave equation in the limit of special relativity. In this limit we obtain the Klein-Gordon equation {2-12, 16} from the Evans wave equation. The classical form of the Klein Gordon equation is Einstein's original definition of relativistic momentum in the special relativistic limit. We then implement standard methods { 16} to find the Newton equation as the limit of special relativity when velocities are small compared to the speed of light, c. The time independent Schrödinger equation is the Newton equation after implementation of the operator equivalence already derived from the equivalence of the Klein Gordon and Einstein equations. The Heisenberg equation (7) is the time independent Schrödinger equation written in a different way but using the same operator definition.

The Klein Gordon equation is the limit in which the Evans wave equation becomes:

$$\left(\square + \frac{m^2 c^2}{\hbar^2} \right) q^a_\mu = 0 \quad - (12)$$

i.e. is the limit represented by:

$$kT = (mc / \hbar)^2 = 1 / \lambda_c^2 \quad - (13)$$

where λ_c is the Compton wavelength and where m is mass. The limit (13) is an example of the principle that equations of general relativity must reduce to equations of special relativity when accelerations (forces) are small in magnitude (free particle limit). This Einsteinian principle dictates that the component of kT originating in rest energy must become the inverse square of the Compton wavelength in the limit of special relativity. The Evans wave equation of motion then describes the motion of a free particle upon which no external forces or accelerations are acting. The modulus of R defined in this limit is the least curvature, and this an example of the Evans principle of least curvature {2-12}. The least, or minimum amount of, curvature needed to define the rest energy of any particle is:

$$R_0 = - \frac{m^2 c^2}{\hbar^2} \quad - (14)$$

and defines the first Casimir invariant { 17} of a particle, its characteristic mass. So spacetime is never everywhere flat. (In an everywhere flat spacetime the universe is empty, there are no particles and no fields, and all energy vanishes identically.) The least curvature

(14) characterises the mass and Compton wavelength of all elementary particles, including the photon, which must therefore have mass in the Evans unified field theory. More precisely, the artificial distinction between particle and field is rejected in favor of radiated and matter fields { 18 }, and there are no point particles, because there are no singularities. Similarly the singularities of quantum electrodynamics and chromodynamics are eliminated in favor of general relativity because there are no point charges. These illustrate some of the major advantages of the new unified field theory {2-12}.

The eigenfunction in the limit represented by (12) is the tetrad..The wave function of the Klein-Gordon equation is a scalar component of the tetrad, a component which is conventionally { 17 } written as the scalar field:

$$\phi = q^0_0 , V^0_1 \ldots , V^4_4 . \qquad - (15)$$

The Evans unified field theory therefore shows that the older concept of scalar field in special relativity must be extended to allow for the fact that ϕ must always be a tetrad component.

Having found the Klein Gordon equation, the time independent Schrödinger equation can be deduced by first deriving the definition of special relativistic momentum in the form:

$$E^2 = p^2 c^2 + m^2 c^4 . \qquad - (16)$$

Here E is conventionally { 16} referred to as the total energy, but is more accurately the total kinetic energy of a free particle, which in special relativity has a component:

$$E_0 = m c^2 \qquad - (17)$$

known as the rest energy. Eq. (16) follows from the Klein-Gordon equation by using the fundamental operator equivalence of quantum mechanics { 17 }:

$$p^{\mu} = i\hbar\,\partial^{\mu}, \qquad -(18)$$

i.e.

$$E = i\hbar\frac{\partial}{\partial t}, \quad \underset{\sim}{p} = -i\hbar\underline{\nabla}. \qquad -(19)$$

We have therefore derived the operator equivalence from general relativity using the Evans wave equation, the reason being that Eq. (16) reduces to the Newton equation in the appropriate limit. In order to derive the Einstein equation (16) and the Newton equation the operator equivalence (18) is needed. In other words if these well known equations are to be deduced from general relativity UNIFIED with quantum mechanics we are led to the operator equivalence (18). Note carefully that in the received opinion the operator equivalence has no justification in general relativity.

Einstein's original definition of relativistic momentum is:

$$p = \gamma m v = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} m v \qquad -(20)$$

and is equivalent to Eq. (18). To see this develop Eq. (20) as follows:

$$p^2 c^2 = \gamma^2 m^2 v^2 c^2 = \gamma^2 m^2 c^4 \left(\frac{v^2}{c^2}\right) \qquad -(21)$$

with:

$$\frac{v^2}{c^2} = 1 - \frac{1}{\gamma^2} \qquad -(22)$$

where

$$\frac{1}{\gamma^2} = 1 - \frac{v^2}{c^2}. \qquad -(23)$$

So Eq. (20) is:

$$p^2 c^2 = \gamma^2 m^2 c^4 \left(1 - \frac{1}{\gamma^2}\right) = \gamma^2 m^2 c^4 - m^2 c^4$$

$$:= E^2 - E_0^2 \qquad -(24)$$

which is Eq. (16).

This means that the Newton law in special relativity is also modified to:

$$\underset{\sim}{F} = \frac{d\underset{\sim}{p}}{dt} = \frac{d}{dt}\left(\gamma m \underline{v}\right). \quad -(25)$$

The kinetic energy in special relativity is calculated from the definition (25) using the work:

$$W_{12} = \int_1^2 \underset{\sim}{F} \cdot d\underset{\sim}{r} = T_2 - T_1. \quad -(26)$$

If we start from rest:

$$W = T = \int \frac{d}{dt}\left(\gamma m \underline{v}\right) \cdot \underline{v}\, dt \quad -(27)$$

and

$$T = m \int_0^v v\, d(\gamma v). \quad -(28)$$

Integrating by parts { 16 }:

$$\begin{aligned} T &= \gamma m v^2 - m \int_0^v \gamma v\, dv \\ &= \gamma m v^2 + mc^2 (1 - v^2/c^2)^{1/2} \Big|_0^v \\ &= \gamma m v^2 + mc^2 (1 - v^2/c^2)^{1/2} - mc^2. \end{aligned} \quad -(29)$$

The special relativistic kinetic energy is therefore:

$$T = mc^2(\gamma - 1). \quad -(30)$$

Therefore the rest energy is this kinetic energy multiplied by $(\gamma-1)^{-1}$ $\cancel{(1-\gamma)^{1/2}}$, showing that the rest energy is kinetic energy and not potential energy.

The Newtonian kinetic energy is now deduced from the equation:

$$T = mc^2\left(1 - \frac{v^2}{c^2}\right)^{-1/2} - mc^2 \quad -(31)$$

using the Maclaurin expansion:

$$\left(1 - \frac{v^2}{c^2}\right)^{-1/2} = 1 + \frac{1}{2}\frac{v^2}{c^2} + \cdots \quad (32)$$

When v << c in Eq. (31) we obtain Newton's kinetic energy for a free particle:

$$T = \frac{1}{2}mv^2 = \frac{p^2}{2m}. \quad -(33)$$

The Schrödinger equation for a free particle is, in consequence:

$$\frac{p^2}{2m}\phi = T\phi \quad -(34)$$

where p is the operator defined by Eq. (19). Using Eq. (19) in Eq. (34) produces the usual form of the time independent Schrödinger equation {1}:

$$-\frac{\hbar^2}{2m}\nabla^2\phi = T\phi. \quad -(35)$$

If there is potential energy V present (as in the harmonic oscillator or Coulomb interaction between proton and electron in an atom) the kinetic energy is written as:

$$T = E - V \quad -(36)$$

where E is the total energy (sum of the kinetic and potential energy). In this case the time independent Schrödinger equation is written as:

$$-\frac{\hbar^2}{2m}\nabla^2\phi = (E - V)\phi. \quad -(37)$$

Eq. (37) is most well known as:

$$\hat{H}\psi = E\psi \quad -(38)$$

where

$$\hat{H} := -\frac{\hbar^2}{2m}\nabla^2 + V \quad -(39)$$

is the hamiltonian operator {1}.

Therefore we have derived the time independent Schrödinger equation from a well defined limit of the generally covariant Evans wave equation without using the spurious and unphysical Heisenberg uncertainty principle. The wave function in the Klein Gordon equation has been identified as a scalar component of the tetrad, and not as a probability. It is well known that the probabilistic interpretation of the wave function of the Klein Gordon equation led to its abandonment in favour of the Dirac equation, and therefore the probabilistic interpretation is untenable. This problem with the Klein Gordon equation is remedied in the Evans unified field theory, which shows that its wave function is a tetrad component (a scalar).A tetrad matrix may have negative scalar components in general, whereas a probability cannot be negative. The Dirac equation may also be derived from the Evans wave equation {2-12} and in the wave function for the Dirac equation is a four component column vector, the Dirac spinor {17}:

$$\psi = \begin{bmatrix} \psi^R \\ \psi^L \end{bmatrix} = \begin{bmatrix} \psi_1^R \\ \psi_2^R \\ \psi_1^L \\ \psi_2^L \end{bmatrix}. \quad -(40)$$

The four components of the Dirac spinor are four components of a tetrad which links two Pauli spinors as follows:

$$\begin{bmatrix} \psi^R \\ \psi^L \end{bmatrix} = \begin{bmatrix} \psi_1^R & \psi_2^R \\ \psi_1^L & \psi_2^L \end{bmatrix} \begin{bmatrix} \psi^1 \\ \psi^2 \end{bmatrix}. \quad -(41)$$

One Pauli spinor (two component column vector) is defined in the orthonormal space:

$$\psi^a = \begin{bmatrix} \psi^R \\ \psi^L \end{bmatrix} \qquad - (42)$$

and the other in the base manifold:

$$\psi^\mu = \begin{bmatrix} \psi^1 \\ \psi^2 \end{bmatrix}. \qquad - (43)$$

So the Dirac spinor contains the four tetrad components arranged in a column vector instead of in the 2 x 2 tetrad matrix defined in Eq. (41). (Strictly speaking the term "tetrad" should be reserved for a 4 x 4 matrix, and the term "diad" used for a 2 x 2 matrix, but the term "tetrad" is conventionally used generically {13}.) The Dirac equation is therefore derived from the Evans wave equation in the free particle limit:

$$\left(\Box + \frac{m^2 c^2}{\hbar^2} \right) \psi = 0 \qquad - (44)$$

and can be considered as four Klein Gordon equations:

$$\begin{aligned} (\Box + m^2 c^2 / \hbar^2)\, \psi^R_1 &= 0 \\ &\vdots \\ (\Box + m^2 c^2 / \hbar^2)\, \psi^L_2 &= 0 . \end{aligned} \qquad - (45)$$

The Dirac equation in the form (44) is equivalent to:

$$\left(\gamma^\mu p_\mu - \frac{mc}{\hbar} \right) \psi = 0 \qquad - (46)$$

which is the form originally derived by Dirac { 17 }. It is well known that the Dirac equation predicts the existence of observable anti-particles in nature, and also predicts the existence of half integral spin, Fermi Dirac statistics, and useful spectral techniques such as ESR, NMR and MRI.

The Evans unified field theory allows the Dirac equation to be identified as a limiting form of the generally covariant Evans wave equation. In the received opinion (the "standard model") this inference is not made, and the standard model is not generally covariant, a major shortcoming remedied by the Evans wave equation.

The Heisenberg equation (6) is clearly seen now as a restatement of the Schrödinger equation (35) using the same operator equivalence (18). Note carefully that we have DERIVED this operator definition from general relativity developed into our unified field theory. The reason why the operator equivalence has been derived is that we have started from the tetrad postulate and Evans Lemma (1), which gives a fundamental wave equation of motion, the Evans wave equation, from the geometrical first principles of general relativity. Historically the operator equivalence was used empirically (i.e. because of the need to explain experimental data {1}) to convert the Newton equation into a wave equation, the Schrödinger equation. So the Evans unified field theory is a major advance in understanding because it derives quantum mechanics from general relativity, an aim of physics throughout the twentieth century.

The Schrödinger equation (35) may also be derived self-consistently from the weak field limit of the Evans wave equation {2-12}. The tetrad q^a_μ is interpreted classically as a gravitational potential or interaction field analogous to the original metric used by Einstein {14}. In the weak field limit the field is assumed to be time-independent and velocities are low compared with c. This means that the relevant component of the tetrad to consider is q^0_0, and that the d'Alembertian becomes the negative of the Laplacian. Therefore:

$$\left(-\nabla^2 + kT\right) q^0_0 = 0, \quad T = \frac{m}{V}. \qquad -(47)$$

In the weak field limit the tetrad component q^0_0 is well approximated by unity, so we recover the Poisson equation for gravitation:

$$\nabla^2 \phi^0 = \frac{k m}{V} \qquad -(48)$$

[16]

which may be expressed as the inverse square law of Newton:

$$\underline{F} = m\underline{g} = \frac{mM}{r^2}\,\underline{k}. \qquad -(49)$$

The principle of equivalence of gravitational and inertial mass means that Eq. (49) also gives the Newtonian kinetic energy (33) for a free particle (upon which no external forces act). In the Newtonian limit the relativistic momentum (20) reduces to mv, where m is the inertial mass and the kinetic energy of the free particle in the Newtonian limit is calculated from the work integral (28) as:

$$T = m\int_0^v v\,dv = \frac{1}{2}mv^2. \qquad -(50)$$

The work integral is in turn calculated from the Newton force equation (second law):

$$\underline{F} = d\underline{p}\,/\,dt \qquad -(51)$$

which is derivable from the Poisson equation (48), thus checking for self consistency. Having derived the Newtonian kinetic energy self consistently from the Evans equation in two complementary ways, the Schrödinger equation follows by applying the operator equivalence (18) to the Newtonian kinetic energy as shown already.

Before proceeding to the derivation of the Heisenberg equation (1) from the rotational form of the Evans field equation {2-12} a few comments are offered as follows on the correct interpretation of the quantities appearing in the wave equation (2), in order to identify and define precisely the kinetic energy, rest energy and potential energy and their associated scalar curvatures.

The total scalar curvature appearing in the Lemma (1) is:

$$R_{total} := -\Box + R \qquad -(52)$$

where the d'Alembertian operator is:

$$\Box = -\frac{1}{\hbar^2} p^\mu p_\mu \qquad -(53)$$

through the operator equivalence (18). So this operator contains the information on the kinetic energy momentum p^μ of a free particle. The rest curvature (14) contains the information on the rest energy of the free particle, whose TOTAL curvature vanishes in the limit of special relativity:

$$R_{total} = \frac{1}{\hbar^2}\left(p^\mu p_\mu - m^2c^2\right) = 0. \qquad -(54)$$

This is what the Einstein equation (16) and its quantized version, the Klein Gordon equation, tells us. In general relativity on the other hand, frames can move arbitrarily with respect to each other, and there are accelerations present, represented by curvatures which are not present for a free particle moving with a constant velocity. These curvatures generalize {13, 14} the concept of force in Newtonian dynamics. The characteristics of field / matter are now represented by the Evans wave equation (2) and its subsidiary geometrical proposition, the Evans Lemma (1). The scalar curvature of the Lemma and wave equation is now made up in general of the rest curvature and curvature due to potential energy:

$$R = -\frac{m^2c^2}{\hbar^2} + R_{potential}. \qquad -(55)$$

The potential curvature arises from interaction between particles and fields, for example the Coulomb interaction within an atom, or the Hooke's law interaction of the harmonic oscillator {1}, or in charge / field interaction in electrodynamics. The generally covariant d'Alembert equation for example, is obtained from the Evans wave equation (2) by incorporating a current term whose origin is potential curvature. The generally covariant Proca equation is the Klein Gordon equation with the generally covariant tetrad potential A^a_μ {2-12} as eigenfunction or wave function. The Proca equation is for the free photon or field, and contains no potential curvature or energy. The latter is introduced through charge current density when there is field / charge interaction. Consideration of the harmonic oscillator potential energy gives rise to zero point energy in the Evans wave equation, both in a dynamical and electrodynamical context. So in general the contracted energy momentum tensor T contains information on potential energy, depending on the type of problem being considered in areas such as dynamics or electrodynamics. For the free particle the rest curvature is defined by:

$$R_0 = -kT_0 \qquad -(56)$$

where T_0 is the relevant part of T for the free particle, i.e. its rest energy density mc^2 / V_0. We therefore arrive at the concept of rest volume of a particle {2-12}, a volume defined by:

$$m_0 V_0 = \frac{\hbar^2 k}{c^2}. \qquad -(57)$$

The existence of rest volume shows that there are no point particles (mathematical singularities) in our unified field theory, and no point particles in general relativity. This leads to a major advantage over the re-normalization procedures inherent in quantum electrodynamics and quantum chromodynamics, while retaining undiminished and perhaps improving on the accuracy of quantum electrodynamics. Re-normalization was rejected by Dirac and other leading thinkers, despite its accuracy. The latter is an unsurprising consequence of the fact that quantum electrodynamics is a perturbation theory. It would be surprising if it were not accurate. It has been shown {19} that O(3) electrodynamics improves on the accuracy of quantum electrodynamics, and it is now known that O(3) electrodynamics is a consequence of our generally covariant unified field theory {2-12}. TOTAL energy in our theory is always conserved, and this is the Noether Theorem. Total charge / current density in our theory is also conserved. These conservation laws are recognized as being equivalent to identities of geometry {2-12}. In Einstein's original theory {14} the Noether Theorem is:

$$D^\mu T_{\mu\nu} = 0 \qquad (58)$$

and the Bianchi identity is:

$$D^\mu G_{\mu\nu} := 0 \qquad (59)$$

where:

$$G_{\mu\nu} = k T_{\mu\nu} \qquad (60)$$

is the Einstein field tensor {13} for classical gravitation. The generally covariant unified field theory of Evans {2-12} extends these considerations self consistently to all radiated and matter fields, and also to quantum mechanics, self consistently including quantum electrodynamics as a generally covariant theory WITHOUT singularities.

We may summarize these considerations through a new concept, total curvature is conserved.

Returning now to our main theme of deriving the Heisenberg equation it is now shown that it is the result of the rotational form of the Evans field equation {2-12}, which was proposed in March 2003 slightly earlier than the wave equation (April 2003). The Heisenberg equation in the form (8) is well known {1} to define angular momentum:

$$\hat{J}_Z = x p_y - y p_x = \pm m_J \hbar \qquad -(61)$$

where $\hat{J}_Z$ is the angular momentum operator and m_J its quantum number, observed for example in atomic and molecular spectra. In our generally covariant theory these rotational quantities are introduced through the torsion form of differential geometry {2-13}, missing from Einstein's original theory of classical gravitation, and angular motion or spin is the spinning of spacetime itself, not the motion of an entity imposed on the reference frame of a flat spacetime. Angular momentum components (orbital or intrinsic) are scalar elements of torsion within a factor $\hbar$. The latter is the minimum quantity of angular momentum in the universe, and so appears for example in the definition (14) of minimum particulate mass (the first Casimir invariant of our theory). The second Casimir invariant is the spin invariant, examples being $\hbar$ itself or the fundamental $\underline{B}^{(3)}$ spin field introduced by Evans {20} in 1992 to describe the inverse Faraday effect. The concept of Casimir invariants of a particle (its mass and spin) were first introduced by Wigner in 1939 {17} for the Poincaré group of special relativity, but in our theory they apply for the Einstein group of general relativity {18}. $\qquad -(62)$

In cylindrical polar coordinates {1}:

$$\hat{J}_Z = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) = -i\hbar\frac{\partial}{\partial \phi}$$

and for a particle on a ring:

$$\hat{J}_Z \psi = \pm m_l \hbar \psi \qquad -(63)$$

which may be interpreted either as a Heisenberg equation or a Schrödinger equation. The torsion form in our theory is defined by the wedge product of tetrads {2-12}:

$$T_{\mu\nu}^{c} := q_\mu^a \wedge q_\nu^b \quad -(64)$$

and is a vector valued two-form anti-symmetric in μ and ν. The definition (64) is true for any base manifold, (i.e. for any type of spacetime with both curvature and torsion present) and so may be written as:

$$T^c := q^a \wedge q^b. \quad -(65)$$

Eq. (65) means that the tetrad is a vector valued one-form that may represent angular momentum within a proportionality factor with the correct units. (The tetrad itself is unitless.) It is therefore possible to define a generally covariant angular momentum operator:

$$\hat{J} q_\mu^a = \pm m_J \hbar q_\mu^a \quad -(66)$$

for any base manifold. Eq. (66) is a rotational form of the Evans field equation {2-12}:

$$\hat{R} q_\mu^a = -kT q_\mu^a \quad -(67)$$

where:

$$\hat{R} q_\mu^a = R q_\mu^a \quad -(68)$$

and

$$\hat{R} = \square. \quad -(69)$$

Eq. (69) is the operator equivalence which allows the Evans wave equation to be obtained from the Evans field equation, thus unifying general relativity with wave (or quantum) mechanics. Eq. (69) therefore gives more insight to the well known operator equivalence (18), which is at the root of the Heisenberg equation.

Consider the solution:

$$\hat{J} q^a_\mu = -m_J \hbar q^a_\mu \quad -(70)$$

and multiply each side of Eq. (70) by angular frequency ω :

$$\omega \hat{J} q^a_\mu = -m_J \hbar \omega q^a_\mu . \quad -(71)$$

The quantum $\hbar\omega$ is the quantum of kinetic energy, and so Eq. (71) may be written as:

$$\hat{T} q^a_\mu = -T q^a_\mu \quad -(72)$$

where

$$T = m_J \frac{\hbar \omega}{c^2 V} . \quad -(73)$$

Eq. (72) can be identified now as the Evans field equation:

$$R q^a_\mu = -kT q^a_\mu \quad -(74)$$

or Evans wave equation:

$$\hat{R} q^a_\mu = \Box q^a_\mu = -kT q^a_\mu \quad -(75)$$

where:

$$\hat{R} = k\omega \frac{\hat{J}}{c^2 V} . \quad -(76)$$

Eq. (76) implies that the quantum of energy $\hbar\omega$ for any particle (including of course the photon, for which it was originally defined by Planck) originates in scalar curvature:

$$R = \pm k m_J \frac{\hbar \omega}{c^2 V} . \quad -(77)$$

The Heisenberg equation has therefore been derived in a second way from our generally covariant theory {2-12} as the non-relativistic quantum limit of Eq. (66).

3. REPLACEMENT OF THE HEISENBERG UNCERTAINTY PRINCIPLE.

In classical general relativity it is always possible to define tetrad matrices with equations such as:

$$x^a = q^a_\mu x^\mu, \quad p^a = q^a_\mu p^\mu \qquad -(78)$$

which correspond {2-12} to:

$$x = 4 q^a_\mu x^\mu_a, \quad p = 4 q^a_\mu p^\mu_a. \qquad -(79)$$

Using the Cartan convention {13}:

$$q^a_\mu q^\mu_a = 1 \qquad -(80)$$

we obtain:

$$x^\mu_a = \frac{1}{4} x q^\mu_a, \quad p^\mu_b = \frac{1}{4} p q^\mu_b. \qquad -(81)$$

So it is always possible to define:

$$J_c = \frac{1}{16} xp\, q_a \wedge q_b. \qquad -(82)$$

In the limit of special relativity, the least curvature principle applies and:

$$J_c \rightarrow \hbar q_c \qquad -(83)$$

so the uncertainty principle is replaced by the classical expression:

$$x_a \wedge p_b \rightarrow \hbar \tau_c. \qquad -(84)$$

Eqn. (82) shows that an angular momentum in general relativity is always the wedge product of tetrads. Eqn. (84) can be written as:

$$x^a \wedge p^b \geq \hbar \tau^c \qquad - (85)$$

and is the law governing the least amount of angular momentum in the limit of special relativity. The essence of our argument is to replace the classical and Euclidean:

$$xp - px = 0 \qquad - (86)$$

by a commutator or wedge product of tetrads (Eq. (85)) in non-Euclidean spacetime with torsion. Such a spacetime was not considered by Einstein {14} in his original theory of gravitation because he restricted attention to Riemannian spacetime with a zero torsion tensor {13}. The tetrads which make up Eq. (85) each obey the tetrad postulate and Evans Lemma and wave equation, so are independently determined in a causal manner.

Heisenberg asserted that complementary observables such as x and p cannot be determined simultaneously to an arbitrarily high precision {1}. In other words complete knowledge of both x and p is impossible. Atkins {1} describes this obscure assertion as follows: "Some pairs of properties are not just simultaneously unknown, they are unknowable." This type of subjective (and therefore arbitrary) assertion was rejected by Einstein, de Broglie, Schrödinger and followers such as Bohm, Vigier and Sachs throughout the twentieth century. Therefore the originators of quantum mechanics and wave particle dualism steadfastly rejected Heisenberg's interpretation.

Using the wave particle dualism of de Broglie in its simplest form {1} we may replace p by $\hbar\kappa$ for ANY particle, not only the photon. This is the essence of Louis de Broglie's great theorem. The wave-number κ also becomes a tetrad in the Evans unified field theory and Eq. (85) means that the commutator of the wave-number and position in general relativity with torsion has a minimum value, the Planck constant $\hbar$. It can now be seen that

Eq. (85) is an extension to angular motion of the phase of a wave, which is conventionally written as the relativistically invariant product $\kappa^{\mu} x_{\mu}$. In the new unified field theory {2-12} the phase is governed by the Evans phase law, essentially a generally covariant Stokes law.

We therefore arrive at the equation:

$$\tau^{c} = q^{a} \wedge q^{b} = \frac{1}{\hbar} x^{a} \wedge p^{b} \quad - (87)$$

for the torsion form, which is missing from Einstein's original theory of general relativity applied to gravitation {14}. The wave particle dualism is expressed in differential geometry by:

$$p^{b} = \hbar \kappa^{b} \quad - (88)$$

and so the torsion form is the wedge product:

$$\tau^{c} = x^{a} \wedge \kappa^{b}. \quad - (89)$$

Similarly, the symmetric metric is the scalar product:

$$g_{\mu\nu} = q_{\mu}^{a} q_{\nu}^{b} \eta_{ab} = x_{\mu}^{a} \kappa_{\nu}^{b} \eta_{ab} \quad - (90)$$

and is therefore identified as a phase tensor. The conventional phase of a wave is defined as the generally covariant scalar $x^{\mu} \kappa_{\mu}$ and is generalized to the Evans phase law {2-12} in the new unified field theory. The Berry phase and other topological phases are examples of the Evans phase law.

The action is in general the tensor valued two form:

$$S_{\mu\nu}^{ab} = \hbar q_{\mu}^{a} q_{\nu}^{b} = \hbar x_{\mu}^{a} \kappa_{\nu}^{b} = x_{\mu}^{a} p_{\nu}^{b} = \frac{1}{c} \int \mathcal{L} \, dx_{\mu}^{a} dx_{\nu}^{b} \quad - (91)$$

where $\mathcal{L}$ is the scalar lagrangian density. The Evans least curvature principle {2-12} unifies the Hamilton least action and Fermat least time principles {1} and implies that the tensor action (91) is minimized in generalized Euler Lagrange equations of motion. Experimentally {1}, the least observable action is $\hbar$, the Planck constant, and this experimental result is expressible in differential geometrical equations such as:

$$|x^a p^b| \geq \hbar\, \delta^{ab}, \quad |x^a \kappa^b| \geq \delta^{ab}, \qquad (92)$$

$$|q^a q^b| \geq \delta^{ab}. \qquad (93)$$

Eqn. (92) for example means that if the wave-number form κ^b becomes very large (high frequency wave) then the position form x^a becomes very small in to keep $\hbar$ a universal constant as observed. Eq.(93) means that the minimum value of the tensor product of tetrads is unity, the everywhere flat-spacetime limit. The Evans wave equation shows that a wave of any kind is a spacetime perturbation, not an entity superimposed on a frame in the flat spacetime limit. The reason for this is that the tetrad is the eigenfunction and that scalar curvature is in consequence itself quantized through the Evans Lemma.

Eq. (92) is the generally covariant expression that replaces the Heisenberg uncertainty principle, and Eq. (93) replaces Born's idea that the product of a wave-function with its complex conjugate is a probability density {1}. These two well known but misguided principles are therefore replaced by straightforward geometrical constraints based on the experimental fact that $\hbar$ is the least observable unit of action and is a universal constant. Every consideration is therefore reduced to geometry as required by general relativity. In the course of this analysis it becomes clear that Einstein and the deterministic School were correct in thinking that quantum mechanics is an incomplete theory. The weaknesses in conventional quantum mechanics are what are conventionally considered its strengths: the uncertainty principle and the Born interpretation of the wave-function. The Evans unified field theory removes these weaknesses and replaces them with equations of differential geometry. The evolution of the wave-function in quantum mechanics is thus recognised as the causal evolution of the tetrad, which is the eigenfunction of a wave equation and therefore characterized by eigenvalues. These eigenvalues are the quanta. The action is in general the tensor valued product of tetrads multiplied by $\hbar$, and therefore the most general tensor valued action form is an object of differential geometry. There is nothing in general relativity which is unknowable, because there is nothing in geometry that is unknowable.

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